DECAY OF AN ARBITRARY DISCONTINUITY ON A CURVILINEAR SURFACE

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The existence of a discontinuous piecewise-analytic solution of the three-dimensional problem of the decay of an arbitrary discontinuity concentrated on a given curvilinear hypersurface at the initial instant is proved for a system of gasdynamics equations.

The problem of the decay of an arbitrary discontinuity on a curvilinear surface occurs in describing the explosion of a nonspherical charge [1], the wave motion of a fluid upon destruction of a dam, etc. These and similar problems of three-dimensional gasdynamics were studied mainly within the framework of approximate models or by using numerical methods. In the planar case, one of the decay configurations of an arbitrary discontinuity was considered in [2] in an exact formulation, where the possibility was shown of constructing the solution in the class of formal power series under definite constraints on the initial data.

1. Formulation of the Problem and the Main Result

Let Γ_0 be an analytic hypersurface in R^3 without reentries separating R^3 into the subdomains D_1 and D_2. At time $t=0$ the state of the gas is given by

$$\mathbf{u}|_{t=0} = \mathbf{u}_i^0(\mathbf{x}), \quad p|_{t=0} = p_i^0(\mathbf{x}), \quad S|_{t=0} = S_i^0(\mathbf{x}), \quad \mathbf{x} \in D_i, \quad i = 1, 2, \tag{1.1}$$

where $\mathbf{u}$ is the gas velocity, p is the pressure, and S is the entropy. The functions $\mathbf{u}_i^0$, p_i^0, S_i^0 are analytic in the domains $\overline{D}_i$, and their limit values are distinct on Γ_0. It is required to describe the gas motion for $t > 0$. The gas is assumed inviscid, non-heat-conducting, and normal [3, 4], and its equations of state are analytic.

The analogous problem has been studied sufficiently well [3, 5] in a one-dimensional formulation. Depending on the initial data on both sides of the discontinuity, three fundamental configurations of the discontinuity occur. According to [3], the formation of a shock wave, a centered wave, and a constant discontinuity is called configuration A, while configuration B corresponds to the formation of two shocks and a contact discontinuity, and configuration C to the formation of two centered waves and a contact discontinuity. Configuration A contains two subcases, the motion of a shock in D_1, a centered wave in D_2, and conversely. In addition to these main configurations, intermediate configurations occur which correspond to the disappearance of the amplitude of one of the waves which hence degenerates into a weak discontinuity on the characteristics.

A piecewise-analytic solution of the three-dimensional problem of the decay of a discontinuity is constructed here. Definite conditions are satisfied on the surfaces of discontinuity of the solution. For weak discontinuities these are conditions of continuous contiguity. For shocks, they are the Hugoniot relationships

$$\begin{gathered}[\rho(u_n - D_n)] = 0, \ [p + \rho(u_n - D_n)^2] = 0, \\ [\varepsilon + p/\rho + {}^1/_2(u_n - D_n)^2] = 0, \ [u_\sigma] = 0\end{gathered} \tag{1.2}$$

and the condition of entropy growth. For contact discontinuities the conditions are

$$[u_n] = 0, \ [p] = 0, \tag{1.3}$$

where [ ] is the symbol of a shock, $u_n = \mathbf{u}\cdot\mathbf{n}$; $\mathbf{u}_\sigma = \mathbf{n}\times\mathbf{u}\times\mathbf{n}$; $\mathbf{n}$ is the normal through the surface of discontinuity by the plane t=const, D_n is the velocity of surface motion in the direction of the normal, ε is the internal energy of the gas, and ρ is the gas density.

One of the elements of the solution will be waves centered on the surface Γ_0 (solutions possessing the property that the acoustic characteristics of one of the families passing through Γ_0 at $t=0$ will cover the domain of definition of the solution). As is shown in [6], to construct a centered wave it is sufficient to give the condition of continuous contact along the characteristic to the known solution and the limit value on Γ_0 of the

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normal velocity component on the second boundary characteristic. As $t \to 0$ the centered wave will behave asymptotically exactly as the simple Riemann wave centered on a plane tangent to Γ_0 at this point.

In this connection, it is clear that the form of the configuration of the decay of the discontinuity is determined locally, in the neighborhood of each point Γ_0, exactly as in the one-dimensional case. Let us introduce the quantities Φ_{ij}, Ψ_{ij} ($i=1, 2, j=1, 2$), Δu_n, Δp as follows:

$$\Phi_{ij} = [(p_j^0 - p_i^0)(\tau_i^0 - \tau_i(p_j^0))]^{1/2}|_{\Gamma_0}, \quad \Delta p = (p_2^0 - p_1^0)|_{\Gamma_0},$$
$$\Psi_{ij} = (\sigma_i(p_j^0) - \sigma_i(p_i^0))|_{\Gamma_0}, \quad \Delta u_n = (u_{n2}^0 - u_{n1}^0)|_{\Gamma_0},$$

where σ is a function of the variables p, S such that $\partial\sigma/\partial p = (\rho c)^{-1}$; $\sigma_i(p) = \sigma(p, S_i^0)$; c is the speed of sound, $\tau_i^0 = (\rho_i^0)^{-1}$; $\tau_i = \tau_i(p)$ is the equation of the Hugoniot adiabat with center at the point τ_i^0, p_i^0. Configuration A occurs upon compliance with the following inequalities:

$$\Psi_{21}\theta(\Delta p) + \Psi_{12}\theta(-\Delta p) < \Delta u_n < \Phi_{21}\theta(-\Delta p) + \Phi_{12}\theta(\Delta p) \tag{1.4}$$

(the normal n is directed into D_1). Configuration B corresponds to the inequality

$$\Delta u_n > \Phi_{21}\theta(-\Delta p) + \Phi_{12}\theta(\Delta p). \tag{1.5}$$

The inequality which gives rise to configuration C when it is satisfied is

$$\Delta u_n < \Psi_{21}\theta(\Delta p) + \Psi_{12}\theta(-\Delta p). \tag{1.6}$$

Reaching equality in one of the inequalities (1.4)-(1.6) corresponds to the intermediate configurations ($\theta(x) = 0$, $x < 0$; $\theta(x) = 1$, $x \geq 0$).

Because of the finiteness of the perturbation propagation velocity, it is sufficient to consider the case of bounded domains D_i adjoining Γ_0. Let the initial data in D_i satisfy the condition: The configuration of the decay of the discontinuity is identical for all points of Γ_0. Then the following theorem is valid.

THEOREM 1. A unique piecewise-analytic solution of problem (1.1) exists in the domain $\Omega = (\overline{D}_1 \cup \overline{D}_2) \times (0, t_0]$ ($t_0 > 0$).

The case of changes in the configuration of the decay in the discontinuity along Γ_0 requires a separate examination in connection with the appearance of new singularities in the solution.

2. Configuration C

In this case the initial data satisfy (1.6). The limit values of the quantity u_n can be found on Γ_0 in the domain outside the centered waves analogously to the one-dimensional case. By the Cauchy–Kovalevskii theorem, analytic solutions exist for the gasdynamics equations taking on the data (1.1) in D_1 and D_2. The centered waves analytic for $t > 0$ [6] are determined by the conditions of adjacency to these solutions and the quantity u_n on Γ_0. There remains to construct a contact discontinuity surface and a solution in the domains bounded by this surface and the boundary characteristics of the centered waves. The analytic data on these characteristics are such that $[u_n] = 0$ and $[p] = 0$ on Γ_0. The existence of the solution of this problem is proved in [7]. The theorem is proved in the case of configuration C and the corresponding intermediate configurations.

3. Configuration B

The inequalities (1.4) are satisfied on Γ_0. Analytic solutions adjoining the data (1.1) are found analogously to the preceding case. It is required to find surfaces of the shocks Γ_1, Γ_2, the surface of the contact discontinuity Γ_3 and the solution of the gasdynamics equations in the domains Ω_1 and Ω_2 bounded by these surfaces (Fig. 1 illustrates the planar case) so that the Hugoniot relationships (1.2) are satisfied on Γ_1, Γ_2, and conditions (1.3) on the contact characteristic Γ_3.

Passage to the new independent variable reduces the problem in a domain with unknown boundaries to a problem in a fixed domain [8]. Let Γ_0 be given parametrically

$$\mathbf{x} = \mathbf{x}_0(\beta, \gamma), \ |\mathbf{x}_{0\beta}| > \delta > 0, \ |\mathbf{x}_{0\gamma}| > \delta > 0,$$
$$|\mathbf{x}_{0\beta} \times \mathbf{x}_{0\gamma}| > \delta > 0.$$

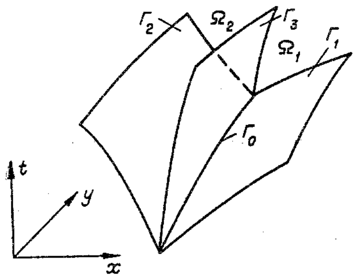

Fig. 1

Let us consider two Cauchy problems in the domain Ω_1:

$$t_\alpha = 1,\ \mathbf{y}_\alpha = d_n(\mathbf{y}_\beta \times \mathbf{y}_\gamma)\cdot|\mathbf{y}_\beta \times \mathbf{y}_\gamma|^{-1},$$
$$t|_{\alpha=0} = 0,\ \mathbf{y}|_{\alpha=0} = \mathbf{x}_0(\beta, \gamma), \tag{3.1}$$
$$t_\tau = 1,\ \mathbf{x}_\tau = u_n(\mathbf{x}_\beta \times \mathbf{x}_\gamma)\cdot|\mathbf{x}_\beta \times \mathbf{x}_\gamma|^{-1},$$
$$t|_{\tau=0} = \alpha,\ \mathbf{x}|_{\tau=0} = \mathbf{y}|_{\tau=0}.$$

The quantity $u_n = \mathbf{u}\cdot\mathbf{n}$ and $\mathbf{n} = (\mathbf{x}_\beta \times \mathbf{x}_\gamma)\cdot|\mathbf{x}_\beta \times \mathbf{x}_\gamma|^{-1}$, the functions d_n will be defined so that at $\tau = 0$ it will agree with the velocity of shock motion in the direction of the normal $\mathbf{n}$. The relations $t = \tau + \alpha$, $\mathbf{x} = \mathbf{x}(\tau, \alpha, \beta, \gamma)$ yield the passage to new variables in the domain Ω_1. For $\tau = 0$ these relations yield the shock surface in parametric form, and for α = const the contact characteristic passing through the shock front at the time $t = \alpha$ [8]. Let us introduce the vector $\mathbf{u}_\sigma = \mathbf{n} \times \mathbf{u} \times \mathbf{n}$ ($\mathbf{u} = u_n\mathbf{n} + \mathbf{n} \times \mathbf{u} \times \mathbf{n}$). The limit values of the velocity $\mathbf{u}$, the pressure p, the entropy s, the velocity of shock front motion in the normal direction d_n on Γ_0 from the domain Ω_1 are determined uniquely from the relations (1.2) and (1.3) as analytic functions of the variables β, γ. Let us convert the gasdynamics equations in the domain Ω_1 to the variables τ, α, β, γ by considering $r = u_n + p/\rho_1 c_1$, $l = u_n - p/\rho_1 c_1$, $\mathbf{u}_\sigma$, and s the desired functions:

$$r_\alpha = (1-m)r_\tau + f_1,\ (1+m)\ l_\tau = l_\alpha + f_2, \tag{3.2}$$
$$\mathbf{u}_{\sigma\tau} = a_1\mathbf{v}_\beta + a_2\mathbf{v}_\gamma,\ s_\tau = a_3 s_\beta + a_4 s_\gamma,$$

where

$$f_1 = \mathbf{a}_5\mathbf{v}_\beta + \mathbf{a}_6\mathbf{v}_\gamma + \mathbf{b}_1\mathbf{v}_\tau + \mathbf{b}_2\mathbf{v}_\alpha;\ m = |d_{n1} - u_{n1}|c_1^{-1};$$
$$f_2 = \mathbf{a}_7\mathbf{v}_\beta + \mathbf{a}_8\mathbf{v}_\gamma + \mathbf{b}_3\mathbf{v}_\tau + \mathbf{b}_4\mathbf{v}_\alpha\ (0 < m < 1);$$

ρ_1, c_1, u_{n1}, d_{n1} are values of the corresponding quantities at a fixed point A_0 ($\beta = \beta_0$, $\gamma = \gamma_0$) of the surface Γ_0, $\mathbf{v}$ is the notation for a vector solution with the components r, l, $\mathbf{u}_\sigma$, s, $\mathbf{x}_\beta$, $\mathbf{x}_\gamma$, $\mathbf{x}_\alpha$; a_i ($i = 1, \ldots, 8$) b_j ($j = 1, \ldots, 4$) are scalar, vector-valued, matrix-valued analytic functions of the argument $\mathbf{v}$. The functions $b_j = 0$ at the point A_0.

By knowing the gas parameters in front of the shock as functions of the variables $\mathbf{x}$, t, the normal to the front and the magnitude of r behind the front, all the gas parameters behind the front can be determined uniquely. We obtain the following boundary conditions for $\tau = 0$:

$$l|_{\tau=0} = (ar + q\ (r, \mathbf{n}, \mathbf{x}, t))|_{\tau=0},\ \mathbf{u}_\sigma|_{\tau=0} = \mathbf{k}\ (\mathbf{n}, \mathbf{x}, t)|_{\tau=0},$$
$$s|_{\tau=0} = w\ (r, \mathbf{n}, \mathbf{x}, t)|_{\tau=0}\ ,\ D_n|_{\tau=0} = d\ (r, \mathbf{n}, \mathbf{x}, t)|_{\tau=0}. \tag{3.3}$$

Let us define the function d_n in Ω_1: $d_n = d(r, \mathbf{n}, \mathbf{x}, t)$; in particular, $d_n = d(r, (\mathbf{y}_\beta \times \mathbf{y}_\gamma)\cdot|\mathbf{y}_\beta \times \mathbf{y}_\gamma|^{-1}, \mathbf{y}, t)$ in (3.1). The function q possesses the property that $q_r = 0$ at the point A_0. The constant a is calculated by the formula

$$a = (-1 + \rho_1 c_1 du_n/dp(A_0))\ (1 + \rho_1 c_1 du_n/dp(A_0))^{-1},$$

where du_n/dp is the angular coefficient of the (p, u_n) shock diagram [3, 4]. Consequently, $du_n/dp > 0$, $|a| < 1$. Extraction of the linear term in r in the first relation of (3.3) is related to the fact that the quantity a plays an important part in the clarification of the problem solvability conditions.

Analogous transformations are performed in the domain Ω_2. The corresponding quantities in Ω_2 are denoted by capital letters $\mathbf{X}$, $\mathbf{Y}$, S, U_n, $\mathbf{U}_\sigma$, etc. The local Riemann invariants are introduced as follows: R =

$U_n - P/\rho_2 c_2$, $L = U_n + P/\beta_2 c_2$ (ρ_2, c_2 are the limit values of ρ and c from Ω_2 at the point A_0). The transformed equations and boundary conditions at $\tau = 0$ have formally the same form as (3.1), (3.2) and (3.3) with the desired functions in lower case replaced by upper case in the notation. The quantity A is given by the formula

$$A = (1 + \rho_2 c_2 dU_n/dP(A_0))(-1 + \rho_2 c_2 dU_n/dP(A_0))^{-1}.$$

Because the slope of the (P, U_n) diagram is $dU_n/dP < 0$, $|A| < 1$. From (1.3) we obtain the boundary conditions for $\alpha = 0$:

$$r = hl + KL,\ R = HL + kl, \tag{3.4}$$

where

$$h = -H = (\rho_1 c_1 - \rho_2 c_2)\ (\rho_1 c_1 + \rho_2 c_2)^{-1};$$
$$K = 2\rho_2 c_2 (\rho_1 c_1 + \rho_2 c_2)^{-1};\ k = 2\rho_1 c_1 (\rho_1 c_1 + \rho_2 c_2)^{-1}.$$

By replacing the desired functions resulting from (3.3), (3.1) and (3.4), and the corresponding relationships in Ω_2, we reduce the boundary conditions to homogeneous conditions. We reduce the nonlinear equations (3.1) to quasilinear equations by continuing them to derivatives (the notation of the transformed quantities is conserved). For $\alpha = 0$ the transformed Riemann invariants satisfy conditions (3.4) and the conditions

$$l = ar,\ L = AR \tag{3.5}$$

for $\tau = 0$. The problem formulated is a Goursat problem (noncharacteristic). The continued equations

$$r_\alpha = (1 - m) r_\tau + f_1,\ (1 + m) l_\tau = l_\alpha + f_2,$$
$$R_\alpha = (1 - M) R_\tau + F_1,\ (1 + M) L_\tau = L_\alpha + F_2 \tag{3.6}$$

are used in a special way to prove its dimensionality. The derivation of the continued equations is rather awkward; hence, we present the result

$$(1 - m)^n (1 - M)^n [\delta_n \Delta_n - kaKA]\, D^n_{n-j,j} r = (1 - M)^n (1 - m)^{n-j} [\delta_n h + kKA] \left[\sum_{i=0}^{n-1} g_i^+ f_1 + \sum_{i=0}^{j-1} a g_i^- f_2 + D^n_{n,0}(l - ar)\right]$$
$$+ K (1 + m)^n (1 - m)^{n-j} \left[\sum_{i=0}^{n-1} (G_i^+ F_1 + A G_i^- F_2) + D^n_{n,0}(L - AR)\right.$$
$$\left. + (1 - M)^n D^n_{0,n}(R - HL - kl)\right] + (1 - M)^n (1 + m)^n \delta_n (1 - m)^{-j} \left[\sum_{i=j}^{n-1} g_i^- f_2 + (1 - m)^n D^n_{0,n}(r - hl - KL)\right],$$

$$(1 - m)^n (1 - M)^n [\delta_n \Delta_n - kaKA]\, D^n_{n-j,j} l = (1 - M)^n (1 + m)^{n-j} \delta_n \tag{3.7}$$
$$\times \left[\sum_{i=0}^{j-1} g_i^+ f_1 + \sum_{i=0}^{n-1} a g_i^- f_2 + D^n_{n,0}(l - ar) + (1 - m)^n D^n_{n,0}(r - hl - KL)\right]$$
$$+ aK (1 + m)^{n-j} (1 - m)^n \left[\sum_{i=0}^{n-1} (G_i^+ F_1 + A G_i^- F_2) + D^n_{n,0}(L - AR)\right.$$
$$\left. + (1 - M)^n D^n_{0,n}(R - HL - kl)\right] + a (1 - M)^n (1 - m)^n (1 + m)^{-j} [\delta_n h + kAK] \left[\sum_{i=j}^{n-1} g_i^+ f_1\right].$$

Moreover, formulas of the form (3.7) with the capital letters replaced by small and small replaced by capitals are valid. Here

$$D^n_{n-j,j} = \partial^n / \partial\alpha^{n-j} \partial\tau^j;\quad g_i^+ = (1 + m)^i D^{n-1}_{n-i-1,i};\quad g_i^- = (1 - m)^i D^{n-1}_{n-i-1,i};$$
$$G_i^+ = (1 + M)^i D^{n-1}_{n-i-1,i};\quad G_i^- = (1 - M)^i D^{n-1}_{n-i-1,i};$$
$$\delta_n = (1 + m)^n (1 - m)^{-n} - ah;\quad \Delta_n = (1 + M)^n (1 - M)^{-n} - AH.$$

The relationships (3.7) possess the property: The total order of differentiation of the unknown functions with respect to τ and α at the point A_0 in the right side is less than **n**. Therefore, they can be used for the successive seeking of the derivatives of the solution. The conditions for solvability of (3.7) have the form

$$\delta_n\Delta_n - kaKA \neq 0,\ n = 1, 2, \ldots \tag{3.8}$$

LEMMA 1. Derivatives of the solution at the point A_0 are determined uniquely.

The inequalities (3.8) follows from the definition of the constants h, H, k, K and the inequalities $0 < m < 1$, $0 < M < 1$, $|a| < 1$, $|A| < 1$, $|h| = |H| < 1$. The assertion is verified easily for first order derivatives.

The fact needed is established by mathematical induction on the n-total order of differentiation with respect to τ and α. Derivatives of order n+1 of the functions r, l, R, L with respect to τ and α are found in terms of derivatives of order n with respect to τ and α from (3.7) and analogous relationships in Ω_2. The derivatives of the other functions are determined afterwards from the differentiated equations (3.1) and (3.2) and their analogs. Because the point A_0 is taken arbitrarily, the assertion is valid for all points Γ_0. Having determined the derivatives, we can construct the solution in the form of formal power series in the variables $\tau, \alpha, \beta-\beta_0, \gamma-\gamma_0$.

Majorants of the series will be constructed in the form of functions of the variable $\eta = \xi(\tau + \alpha) + \beta - \beta_0 + \gamma - \gamma_0$ ($\xi > 1$ is a constant) in order to prove the convergence. The equations for the majorants l_m, L_m, r_m, R_m, s_m, S_m, etc. occur upon replacing the coefficients of the quasilinear system by their majorizing functions. The property that certain coefficients of the system vanish at the point A_0 is retained here. The dependence of the coefficients on $\tau, \alpha, \beta, \gamma$ is majorized by a dependence on η. Let us examine the case when $a \neq 0$, $A \neq 0$ (e.g., the quantities a and A can possibly vanish in a polytropic gas with adiabatic index $\varkappa$: $5/3 \leq \varkappa \leq 2$). Let us demand that the majorants satsify the relationships

$$l_m = |a|r_m,\ L_m = |A|R_m,\ \ R_m = |H|L_m + k_0 l_m,\ r_m = |h|l_m + KL_m, \tag{3.9}$$

where

$$k_0 = |aA|^{-1}K^{-1}(1 - |ah|)(1 - |AH|).$$

The latter relationship assures the existence of a nontrivial solution of the linear homogeneous equations (3.9). We seek the solution of the majorant equations in the form of functions of the variable η. The majorant equations corresponding to (3.6) have the form

$$\xi r_m' = m^{-1}f_{1m},\ \ \xi l_m' = m^{-1}f_{2m},\ \ \xi R_m' = M^{-1}F_{1m},\ \ \xi L_m' = M^{-1}F_{2m}. \tag{3.10}$$

Because of (3.9) the right sides of (3.10) are connected by the relations

$$f_{2m} = |a|f_{1m},\ F_{1m} = Mm^{-1}k_0|a|(1 - |AH|)^{-1}f_{1m},\ \ F_{2m} = |A|F_{1m}. \tag{3.11}$$

The initial conditions for all the majorants is that they vanish at the point A_0. Then (3.9) follows from (3.10) and (3.11). The majorant f_{1m} is selected so that ($\gg$ is the majorizing relation)

$$\begin{gathered} f_{1m} \gg K_1 f_1,\ f_{1m} \gg |a|^{-1}K_1 f_2, \\ f_{1m} \gg mK_1(1 - |AH|)(Mk_0|a|)^{-1}F_1, \\ f_{1m} \gg m(1 - |AH|)K_1(Mk_0|aA|)^{-1}F_2\ (K_1 = \max(1, kk_0^{-1})). \end{gathered} \tag{3.12}$$

For instance, it is sufficient to take the sum of the majorants f_i and F_i (i=1, 2) with a sufficiently large numerical coefficient as f_{1m}. The system of ordinary equations to determine the majorant is reduced to normal forn if ξ is selected sufficiently large and the existence of the solutio of the system follows from the Cauchy–Kovalevskii theorem. The appropriate calculations are analogous to those performed in [8]. By construction, the functions found satisfy the conditions (3.4) and (3.5) with k replaced by k_0. A formula of the form (3.7) can be obtained for them because of the equations. The fact that the constructed analytic functions majorize the formal power series of the solution of the problem results from these formulas, the properties (3.12), and the property of majorizability of the remaining equations of the system. Convergence is proved for $a \neq 0$, $A \neq 0$.

Let A=0. Then instead of (3.9), we demand compliance with the relationships

$$\begin{gathered} l_m = |a|K(1 - |ah|)^{-1}L_m,\ r_m = K(1 - |ah|)^{-1}L_m, \\ R_m = (|H| + |a|kK(1 - |ah|)^{-1})\,L_m. \end{gathered} \tag{3.13}$$

If A = 0, a = 0, in place of (3.9), we staisfy the relationships

$$R_m = |H|L_m + kl_m, \ r_m = |h|l_m + KL_m.$$

The subsequent considerations are analogous to those in the preceding case.

The possibility of inverting the mapping $t = \tau + \alpha$, $\mathbf{x} = \mathbf{x}(\tau, \alpha, \beta, \gamma)$ for small t is proved exactly as in [8]. The assertion of Theorem 1 is proved in the case (1.5).

4. Configuration A

The initial data (1.1) on Γ_0 satisfy the inequalities (1.4). As in the preceding case, the first step is to construct analytic functions taking on the data (1.1), in D_1 and D_2. From the relationships of the strong discontinuity and the relationships of a central wave [6] the limit values of the gas parameters can be determined on Γ_0 from the domain Ω_1 behind the shock, and the domain Ω_2 in a central wave. The condition of contact with the known solution and data on Γ_0 is sufficient for construction of the central wave. The problem reduces to seeking the shock surface Γ_1, the contact discontinuity surface Γ_3 and the solution in the domains Ω_1, Ω_2 which satisfies the conditions (1.2) on Γ_1, (1.3) on Γ_3, and the conditions of continuous contact with the given solution on the known characteristics Γ_2. The intermediate configurations, corresponding to configuration A, reduce to the same problem.

The same transformations are satisfied in the domain Ω_1. New variables are introduced in the domain Ω_2 as follows: The known surface Γ_2 is given parametrically by the equations $t = \alpha$, $\mathbf{x} = \mathbf{x}_1(\alpha, \beta, \gamma)$, where the function $\mathbf{x}_1(\alpha, \beta, \gamma)$ is the solution of the problem

$$\mathbf{x}_{1\alpha} = (U_n - C)(\mathbf{x}_{1\beta} \times \mathbf{x}_{1\gamma}) \cdot |\mathbf{x}_{1\beta} \times \mathbf{x}_{1\gamma}|^{-1}, \ \mathbf{x}_1|_{\alpha=0} = \mathbf{x}_0(\beta, \gamma).$$

Furthermore, the Cauchy problem

$$T_\tau = 1, \ \mathbf{X}_\tau = U_n(\mathbf{X}_\beta \times \mathbf{X}_\gamma) \cdot |\mathbf{X}_\beta \times \mathbf{X}_\gamma|^{-1},$$
$$\mathbf{X}|_{\tau=0} = \mathbf{x}_1(\alpha, \beta, \gamma), \ T|_{\tau=0} = \alpha$$

is solved. The relationships $T = \tau + \alpha$, $\mathbf{X} = \mathbf{X}(\tau, \alpha, \beta, \gamma)$ yield the passage to $\tau, \alpha, \beta, \gamma$. The Riemann invariants R and L are introduced exactly as in Sec. 3. The transformed gasdynamics equations formally have the form (3.2) (with small letters replaced by capitals in the notation) with the sole difference that M = 1. Given on the boundary $\tau = 0$ are L, $\mathbf{U}_\sigma$, S as functions of the variables $\mathbf{X}$, T. Therefore, the problem under consideration is analogous to the one studied in Sec. 3 in the case A = 0, M = 1. In particular, the condition of formal solvability

$$(1 + m)^n(1 - m)^{-n} - ah \neq 0, \ n = 1, 2, \ldots$$

is satisfied. Further discussion is analogous to Sec. 3.

Proof of the possibility of inverting the mapping $t = \tau + \alpha$, $\mathbf{x} = \mathbf{x}(\tau, \alpha, \beta, \gamma)$ in the domains Ω_1 and Ω_2 is presented in [7, 8]. The theorem is proved.

The method elucidated for the construction of the solution can be used for an approximate computation of the three-dimensional decay of a discontinuity in the neighborhood of the initial surface of discontinuity.

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DETONATION-GENERATED SHOCK WAVE

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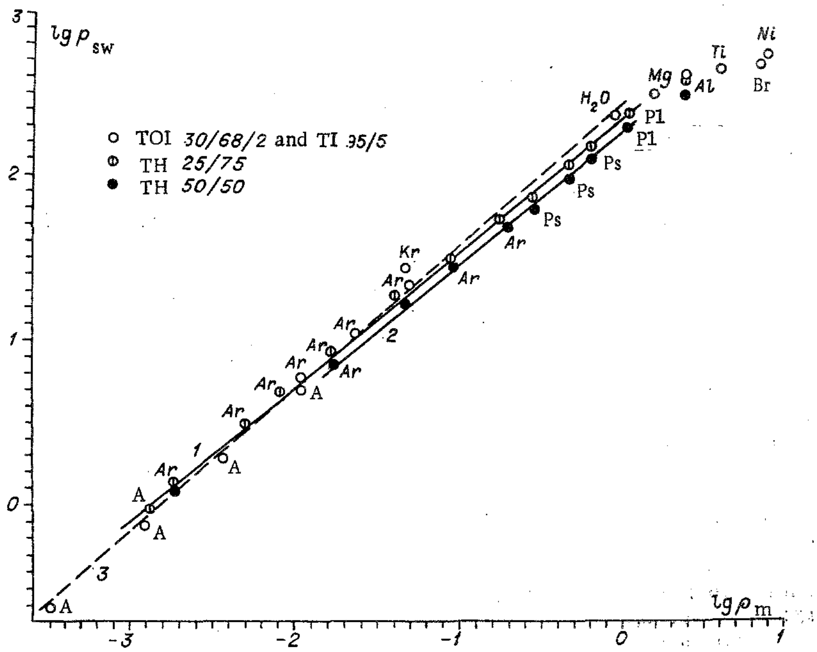
Let us consider a cylindrical explosive charge of sufficiently large dimensions in which a plane detonation wave traveling along the axis is initiated. When this wave emerges at the charge endface, decay of the discontinuity occurs. Let a condensed explosive charge be in contact with an inert medium of lower dynamic stiffness (gas, water, organic material). Then a shock will appear in the inert medium, and inversely in the explosion products (EP), an unloading wave with two weak discontinuities.

There are some experiments of similar type in which the detonation and shock wave parameters have been measured. Processing the experiments using explosions of a trotyl-hexogene (TH) [1] and a trotyl-octogene-inert (TOI) mixture [2], clarified an interesting regularity.

Plotted along the axes in Fig. 1 are logarithms of the shock wave pressure p_{sw} and the initial density of the inert medium ρ_m in which this wave emerged (the letters denote the composition of the inert medium, including A for air, Ps for polystyrene, Pl for Plexiglas, Br for brass, and the numbers 1-3 are numbers of the corresponding equations. All the experimental points for an explosive of definite composition and initial density ρ_0 lie on a line independently of the composition of the inert medium if an unloading wave returns backward in the EP.



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